Załącznik 3

1. **Name and last name: Gniewomir Sarbicki**

2. **Certificates and scientific degrees**

    a) M. Sci degree: June 2004, physics, Nicolaus Copernicus University, M. Sci thesis: *Geometrical description of three qubits* (supervisor: Prof. Dr A. Jamiołkowski)

    b) M. Sci degree: October 2005, mathematics, Nicolaus Copernicus University, M. Sci thesis: *Periodic solutions of a coupled pendulum* (supervisor: Prof. Dr S. Rybicki)

    c) PhD: April 2009, physics, Nicolaus Copernicus University, PhD thesis: *Geometrical properties of the sets of separable quantum states and entanglement witnesses* (supervisor: Prof. Dr A. Jamiołkowski)

3. **Academic appointments**

    a) 2008-2011 r.

    Research Assistant, Department of Physics, Astronomy and Informatics, Nicolaus Coopernicus University

    b) 2011-2012 r.

    Postdoc, Stockholms Universitet

    c) 2012-2015 r.

    Assistant Professor (Adiunkt), Department of Physics, Astronomy and Informatics, Nicolaus Coopernicus University

4. **Awards**

    - Scholarship of the Ministry of Science (2001)
    - Scholarship "A step into future" for PhD students (2008)
    - Team Award of second degree of the Rector of Nicolaus Copernicus University (2012)
    - Team Award of second degree of the Rector of Nicolaus Copernicus University (2013)

5. **Achievement indicated in accordance with the Art. 16 Item 2 of the Act of 16 March 2003 on Academic Degrees and Title and Degrees and Title in Art (Journal of Laws No. 65, item. 595 as amended):**

    a) Title of the achievement *Geometrical properties of cones in quantum mechanics - optimality and exposedness of entanglement witnesses*

    b) List of publications included in the achievment

[W1] Dariusz Chruscinski, Justyna Pytel, Gniewomir Sarbicki *Constructing new optimal entanglement witnesses* Phys. Rev. A 80, 062314 (2009)
The estimated own contribution: 30%, my contribution was proving the positivity of maps used to construct the witnesses

[W2] Remigiusz Augusiak, Gniewomir Sarbicki, Maciej Lewenstein *Optimal decomposable witnesses without the spanning property* Physical Review A 84, 052323 (2011)
The estimated own contribution: 25%, my contribution was generalising the example from 3×3 *to* $n \times m$ *and contributing in the paper editing.*

[W3] Dariusz Chruściński, Gniewomir Sarbicki *Exposed positive maps: a sufficient condition* J. Phys. A: Math. Theor. 45 (2012) 115304
The estimated own contribution: 40%, my contribution was applying the criterion to the maps coming from transposition.

[W4] Gniewomir Sarbicki, Dariusz Chruściński *A class of exposed indecomposable positive maps* J. Phys. A: Math. Theor. 46 015306 arXiv:1201.5995 (2013)
The estimated own contribution: 80%, my contribution was applying the exposedness criterion to this class of maps.

[W5] Dariusz Chruściński, Gniewomir Sarbicki *Exposed positive maps in* $M_4(\mathbb{C})$ Open Sys. Inf. Dyn. 19 (3) 1250017 (2012)
The estimated own contribution: 50%, my contribution was applying the exposedness criterion to the maps.

[W6] D.Chruściński, G.Sarbicki *Optimal entanglement witnesses for two qutrits* Open Sys. Information Dyn. 20 (2) 1350006 (2013)
The estimated own contribution: 50%, my contribution was finding the general form for the spanning vectors in this family of witnesses.

[W7] Dariusz Chruściński, Gniewomir Sarbicki *Disproving the conjecture on structural physical approximation to optimal decomposable entanglement witnesses* J. Phys. A: Math. Theor. 47, 195301 (2014)
The estimated own contribution: 50%, my contribution was applying the realignment criterion to the proposed class and the analytical proof of breaking the criterion.

[W8] Dariusz Chruściński, Gniewomir Sarbicki *Entanglement witnesses: construction, analysis and classification* J. Phys. A: Math. Theor. 47, 483001 (2014)
The estimated own contribution: 20%, my contribution was the fragments about geometry of cones and about Bell inequalities.

[W9] A.Rutkowski, G.Sarbicki, D.Chruściński *A class of bistochastic positive optimal maps in* $M_d(\mathbb{C})$ Open Sys. Inf. Dyn. 22 (3) 1550016 (2015)
The estimated own contribution: 30%, my contribution was proving the optimality of maps.

c) The description of scientific goals and results of the above publications with a discussion of their potential applications.

In the rest of this report the references [W1]-[W9] refer to the papers included in the scientific achievement underlying the habilitation request.

Geometrical introduction

Sets of states and sets of observables having interesting information properties are in most cases *cones* or differences of cones. A cone K is any closed. convex subset of a real vector space, factorised by halflines ($x \in K \Rightarrow \forall\lambda \in \mathbb{R}_+ \cup \{0\}\ \lambda x \in K$). A halfline passing through the point x is called a *ray* of the point x. Any cone is closed on taking linear combinations of its elements, with positive coefficients.

In a cone one can distinguish a lattice of subsets called *faces*. A face F is the subset of a cone with the property, that taking arbitrary $x, y \in K$, if $x \in F\ \wedge x - y \in K$, then $y \in F$. Faces are closed with respect to taking linear combinations of their elements, with positive coefficients, so they are cones. A intersection of two faces is again a face, so one can define the minimal face containing a given element and we call it the *face generated by the element*.

Assume, that we decompose an element x of a cone K as a linear combination of other elements of the cone, with positive coefficients. If only such decompositions are decompositions into elements of the ray of the point x, then we call x an *extremal point* in K.

For a cone K in space X one can consider a subset of X^* of all forms taking non-negative values on elements of K. This set turns out to be a cone in X^* and we call it a cone dual to K and denote as K^*. In our considerations we will restrict ourselves to the case when $X = X^{**}$, thus we have $K^{**} = K$.

For an element x of a cone K one can ask which elements of K^* annihilate it. It turns out, that they constitute a face in K^* called the *face dual* to the element x. Moreover, two elements generating the same face have the same dual face, thus the duality maps the lattice of faces of the cone K into the lattice of faces of the cone K^*. Let us denote this map by Φ. In the same way one constructs the map Φ^* prescribing faces of the cone $K^{**} = K$to the faces of the cone K^*. The map $\Phi^*\Phi$ prescribes in general to a face of K some of its superfaces. The faces mapped in such operation to themselves are called *exposed faces*. In particular, one-dimensional exposed faces are called *exposed rays* and elements of such rays are called *exposed points*.

Subsets of sets and observables being the matter of interest

We will consider a set of normal quantum states (density operators). They are selfadjoint, semipositive definite trace-class operators of trace equal one. For convienience we drop the trace normalisation condition and the matter of interest is the cone of traceless, selfadjoint, semipositive definite operators.

If in a quantum system one can distinguish two subsystems, let's call them A and B, then the Hilbert space of the system is a tensor product of Hilbert spaces of subsystems $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$. For some states of the system there exist decompositions of the form $\rho = \sum_i \rho_i^A \otimes \rho_i^B$, where $\rho_i^A \in \mathbb{B}_1(\mathcal{H}_A)$, $\rho_i^B \in \mathbb{B}_1(\mathcal{H}_B)$ are states respectively of subsystems A and B [1] (although this decomposition doesn't have to be unique). These distinguished states are called *separable states*. They form a cone being a subcone of the cone of all states. The states in the difference of these two cones are called *entangled states*. An arbitrary classical state of a composed system can be constructed as a statistical mixture of product states. By definition it is impossible to construct in such a way an entangled state, so the correlations between subsystems in such states are non-classical. Such correlations are used in a number of quantum information protocols (teleportation, cryptography), the are also a computational resource for quantum computers.

Usually, with possible applications in view, we are interested in entanglement between spatially separated subsystems. One gets an entangled state of two particles, one being in a place A, the other being in a place B, from a source of particle pairs. We want to check the entanglement between particles.

For a given state, deciding if it is separable or entangled is hard [2, 3]. It is equivalent to checking, whether for the investigated state the expected value of all observables positive on pure product states is non-negative [4]. Such observables form a cone, which will be denoted further as $\mathcal{W}_1$, which is a supercone of the cone of semi-positive definite observables ($\mathcal{B}_+$). By definition the cone $\mathcal{W}_1$ is dual to the cone $\mathcal{B}_1$.

Elements of the set $\mathcal{W}_1 \setminus \mathcal{B}_+$ (observables positive on pure product states, but not semipositive definite) are called entanglement witnesses. If the expected value of a witness in a given state is negative, then we say that it is an entangled state detected by the witness. To any witness one can prescribe the set of states it detects.

There are several separability criteria, which allow to detect entanglement for explicitly given matrices of a composed system. Among the most important are partial transposition criterion and realignment criterion [6] [1] . They refer to whole families of entanglement witnesses. In practice their use require the knowledge of all matrix elements of the density operator (the full state tomography) with big accuracy and big significance level. It requires a large number of measurements to be done. Often, we know a lot about the state from the physics of the source, so we can construct a witness, the negative value of which will assure us about the entanglement between particles coming from the source (and confirm the usability of such a pair in quantum protocols). The numeric value of the expectation value of a (appropriately normalised) witness can also be a measure of quality of entanglement. An estimation of the expectation value of an observable with desired accuracy is realised by means of multiple measurements and averaging. It is a multiple measurement using the same measuring device settings. It is worth to mention, that since a witness is an observable on the system consisting of two spatially separated subsystems, there is no measuring device realising it. It should be decomposed into a linear combination of tensor products of local observables and a multiple measurement should be performed on them. In this way we reconstruct an expectation value of a witness. One proceeds in the same way when measuring the breaking of a Bell inequality. It is not surprising - any Bell inequality can be translated to an entanglement witnesses, more precisely it refers to a whole class of entanglement witnesses referring to different choices of local observables in the Bell inequality (such an equivalence class is called the *Device Independent Witness*), [W8] and references there in).

The partial transpose criterion plays a distinguished role - for a long time the only states constructed and detected experimentally were the states detected by this criterion. Entangled states not detected by partial transposition criterion, called PPT entangled states have interesting physical properties. Their entanglement is called the bound entanglement (whether there exist non-PPT bound entangled states is still an open question, the examples are still not known). PPT states also constitute a cone, denoted by $\mathcal{B}_{PPT}$, being a section of the cone $\mathcal{B}_+$ and its image in the partial transposition map. The cone dual to $\mathcal{B}_{PPT}$ is a convex hull of the cone $\mathcal{W}_1$ and its image in the partial transposition map. We will denote it by $\mathcal{W}_D$. Its non-semipositive elements are called *decomposable witnesses*. We call witnesses from the set $\mathcal{W}_1 \setminus \mathcal{W}_D$ *non-decomposable witnesses*.

[1] In practice, checking for the entanglement of a state numerically one uses an algorithm based on the Doherty criterion [7, 8], but proving the entanglement in classes of states analytically, one uses the partial transposition criterion, and for the states not detected by this criterion (called PPT states) in the generic case one uses the realignment criterion and its amplifications

Optimality and spanning

In the cone of witnesses one can distinguish optimal elements, which detect maximal (in the sense of inclusion) sets of entangled states. From the point of view of applications, there is no need to use non-optimal witnesses - by optimising a non-optimal witness we do not lose already detected states but we broaden the range of applicability of our tool.

An equivalent optimality definition is that the witness after subtracting any semi-positive observable is no longer a witness [9].

There is a simple sufficient condition for optimality of entanglement witness - if the set of vectors $\{\alpha \otimes \beta : \langle \alpha \otimes \beta | W | \alpha \otimes \beta \rangle = 0\}$ spans the whole Hilbert space of the system, then the witness W is optimal [9]. We call such W *the spanning witnesses* or *the one with spanning property.*

We have used the spanning criterion to prove the optimality of classes of witnesses [W1], [W9]. In the work [W2] we construct a decomposable, optimal witness without spanning property in arbitrary dimensions of subsystems, greater then 2. We prove its optimality directly from the definition.

The paper [9] which introduces the definitions of optimality and spanning defines also ND-optimality, known in subsequent literature as *bioptimality.* The non-decomposable witness is ND-optimal if no other witness detects more (in the sense of inclusion) entangled PPT states. It turns out to be equivalent to the property, that the entanglement witness after subtracting any decomposable witness in no longer an entanglement witness. The sufficient condition for ND-optimality is that the witness as well as its partial transposition have spanning property.

In the paper [W10] I have formulated the geometrical interpretation of optimality. The optimality can be defined for arbitrary two cones K i L fulfilling the inclusion relation $K \subset L$ - an element of $L \setminus K$ is optimal if and only if no other element of $L \setminus K$ detects more elements of $K^* \setminus L^*$ (in the sense of inclusion). In case of cones $\mathcal{B}_+$ and $\mathcal{W}_1$ one gets the standard definition of optimality. Taking the cones $\mathcal{W}_D$ and $\mathcal{W}_1$ one gets ND-optimality. In the general case one can prove the equivalent definition of optimality - an element $w \in L \setminus K$ is optimal if and only iff $\forall k \in K\ \{0\}, w - k \notin L$.

Optimality is a property of entire faces of the cone L. I have proven, that an element is optimal if and only if the smallest face containing it contains no element of the subcone K. Nowadays the face definition of optimality is given as the primary one [10].

In a similar manner one can characterise the set of spanning elements - an element has the spanning property if the smallest exposed face containing it contains no elements of the subcone K. The general spanning criterion for an arbitrary pair of states implies, as its special case spanning criteria for the standard optimality and ND-optimality.

The geometrical characterisation of optimality and spanning gives answers the question when are these two conditions equivalent. It happens when the bigger cone has all its faces exposed. The cone $\mathcal{W}_1$ does not posess this property and the witness coming via Jamiołkowski isomorphism from the Choi map is an example of an optimal witness without the spanning property. Cones with all faces exposed are e.g. self-dual cones and polyhedral cones, arising as sections of a finite number of halfspaces.

The geometrical characterisation of optimality and spanning allows to formulate these concepts for an arbitrary pair of cones. In the quantum mechanics of many particles it can be the detection of various types of entanglement, in cones of conditional probabilities considered in the quantum steering theory, the detection of non-quantum correlations among non-informing correlations and so on.

Exposedness

If we do not ask about a possibility to improve an existing experiment, but rather about the minimal set of witnesses detecting any entanglement, then we ask about the extremal elements in the cone of witnesses - the elements, which cannot be decomposed as convex combination of any other two elements of the cone, not being in the same ray. Among them we have the set of exposed points - lying on rays which are intersections of the cone with a tangent hypersurface. The Straszewicz theorem [11] states that the set of exposed elements is a dense subset of the set of extremal elements. It implies (by continuity), that if an entangled state is detected by an extremal witness, then it is detected also by an exposed witness, close to the former one. Exposed witnesses are able to detect entanglement of any state.

In the paper [W3] we have formulated an exposedness criterion, similar to the spanning criterion for optimality - if the set of vectors $\{\alpha^* \otimes \alpha \otimes \beta : \langle \alpha \otimes \beta | W | \alpha \otimes \beta \rangle = 0\}$ spans the space of dimension $d_1^2 d_2 - d_1$ (we have called this property *the strong spanning property*) and in addition the map related to the witness via Jamiołkowski isomorphism is irreducible, then the witness is exposed. Using this criterion we were able to prove the exposedness (and then the extremality as well) for a class of witnesses in even dimensions [W4] which earlier were only known to be optimal [W5].

This criterion is relatively weak, i.e. it is easy to give a construction of an exposed witness without the strong spanning property. In the paper [W3] we show, that all partial transposes of projectors, which are exposed [12] have no property of strong spanning if the dimensions of subsystems are not equal.

The SPA hypothesis

In the paper [13] P.Horodecki and A.Ekert have formulated the SPA hypothesis about structural physical approximations of entanglement witnesses. The $SPA(W)$ is defined as a state $SPA(W) = W + \lambda\rho_0$, where $\lambda = \min\{\lambda : W + \lambda I \geqslant 0\}$ and ρ_0 is the maximally mixed state. The SPA hypothesis states, that for optimal witnesses the $SPA(W)$ is always a separable state. The hypothesis is pupported by a large range of known witnesses [14]. It has serious practical implications for the procedures of measuring of optimal witnesses[14]. The paper [15] gives a non-decomposable witness being a counterexample to the hypothesis and the hypothesis has been weakened to the case of decomposable witnesses. In the paper [W7] we have shown a family of decomposable witnesses in $\mathbb{C}^3 \otimes \mathbb{C}^3$ which break the hypothesis, so its weakened version also turns out to be false. Via embedding, the family is a counterexample for the hypothesis for any subsystems of dimensions greater than 2. The hypothesis has been proven in dimensions $2 \times n, n \leqslant 4$, so it remains to be checked for $2 \times n, n > 4$.

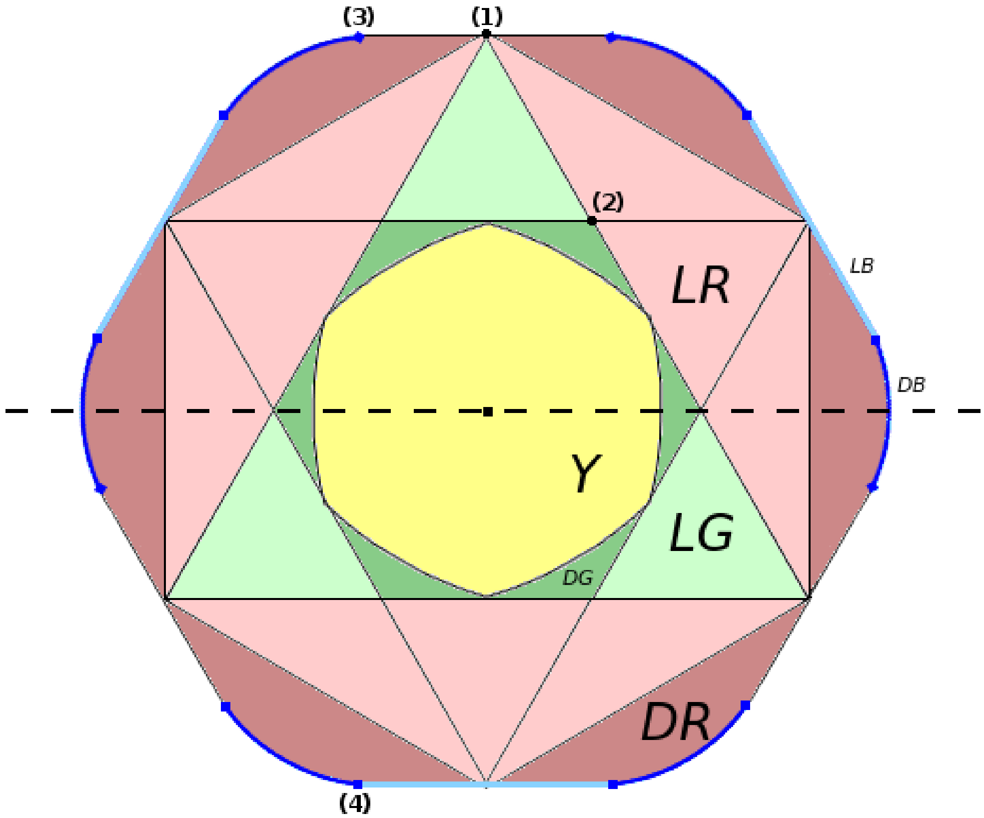
Graphical summary

The above picture from the paper [W8] is an ilustration of the presented issues by means of cones in $\mathbb{R}^3$. The picture presents a section of $\mathbb{R}^3$ by the surface of the picture.

The partial transposition in $\mathcal{B}_H(\mathbb{C}^3)$ is a linear involution and is represented in the picture by the reflection. The surface of the reflection (the set of fixed points of the partial transposition) is represented by the dashed line.

The light-green (LG) triangle represents the set of semi-positive definite operators (states). The point (1) is a pure state. The intersection of the set of states with its reflection is the set of PPT states, represented as the hexagon on the picture. Its subset, the yellow (Y) "pumped" hexagon is the set of separable states. The dark-green (DG) regions of the hexagon of PPT states are entangled PPT states. The point (2) is an extremal PPT state and the so-called edge-state (edge-state - subtracting from it any separable state leads to an operator with negative partial transposition, according to the general theory of detection it is an element optimally detecting decomposable witnesses in the set of all witnesses).

The convex hull of the sum of the set of positive operators and the image of this set after partial transposition forms a cone $\mathcal{W}_D$ (decomposable witnesses and positive observables). It is represented on the picture as the big hexagon. Its light-red (LR) part represents the set of decomposable witnesses. The cone dual to the cone of separable states (yellow (Y) area) is the cone $\mathcal{W}_1$. It is the biggest figure on the picture. The dark-red (DR) area represents the set of non-decomposable witnesses.



On the boundary of the set of witnesses one can distinguish the extremal witnesses - closed dark-blue (DB) arcs in the picture. Among them only the points from the open arcs are exposed, the point (4) is an extremal but not exposed witness (like for example the witness related to the Choi map). The light-blue (LB) points on the boundary represent the optimal witnesses which are not exposed. In the centres of the light-blue segments lie the optimal decomposable witnesses. The ND-optimal witnesses in the picture are the extremal ones (in fact the set of ND-optimal witnesses is the superset of the set of extremal witnesses).

The SPA hypothesis is fulfilled in the picture only by the optimal decomposable witnesses. In fact, among optimal decomposable as well as among optimal non-decomposable witnesses there are both examples supporting the SPA hypothesis and its counterexamples (when the dimension of subsystems are greater than 2).

6. **Other Scientific achievements**

a) List of publications not included in the achievment

[W10] Gniewomir Sarbicki *General theory of detection and optimality* arXiv:0905.0778 (2009)

[W11] G. Sarbicki *Geometry of Local Orbits in Three Qubit Problem* Open Systems & Information Dynamics 12: 143-161 (2005)

[W12] G. Sarbicki *Spectral Properties of Entanglement Witnesses* J. Phys. A: Math. Theor. 41 (2008) 375303

[W13] G. Sarbicki *Spectral Properties of Entanglement Witnesses and Separable States Journal* of Physics: Conference Series 104 (2008) 012009

[W14] D. Chruściński, A. Kossakowski, G. Sarbicki *Spectral conditions for entanglement witnesses vs. bound entanglement* Phys. Rev A 80, 042314 (2009)

[W15] G. Sarbicki, I. Bengtsson *Dissecting the qutrit* J. Phys. A: Math. Theor. 46 035306 (2013) arXiv:1208.2118

[W16] F. A. Wudarski, P. Należyty, G. Sarbicki, D. Chruściński *On admissible memory kernels for random unitary qubit evolution* Phys. Rev. A 91, 042105 (2015) arXiv:1501.05179

b) Characterisation of main research results not included in the achievement

Orbits of group of local operations

The subject of the research was the topology of orbits of the group of local operation for two and three qubits. Orbits are classified with respect to the subgroups being their stabilisers (a isotropy subgroup). Identifying in the group $SU(2)^{\times 3}$ the elements differing by the stabiliser, one gets a set homeomorphic to the orbit. In general the orbits have a topology of fibre bundles and in all cases I have found their base spaces and fibres.

Spectral properties of entanglement witnesses and separable states

Using the methods of algebraic geometry I have found a condition for maximal number of negative eigenvalues of an entanglement witness and more general for a k-Schmidt witness ([16] - an observable not semi-positive definite, but semi-positive on all vectors of Schmidt rank not exceeding k). I have proven the sufficient conditions which have to be satisfied by the kernel and the subspaces V_+ and V_- spanned by eigenvectors related to positive and negative eigenvalues respectively. The stronger condition allows to guarantee, that having given two positive observables W_+ and W_- supported respectively on V_+ and V_-, there exists a sufficiently large λ such that $\lambda W_+ - W_-$ is an entanglement witness (all proofs are derived for the general case of k-Schmidt witnesses). Introducing the Schmidt coefficient for a subspace as the maximal highest Schmidt coefficient of normalised vectors in this subspace, I have formulated for a given orthonormal basis, whose eigenvectors satisfy the necessary conditions, a quantitative conditions for eigenvalues of an entanglement witness diagonal in the given basis.

In the paper [W13] I define the $S-sup$ norm, which is an analogon of the supremum norm, but the supremum is taken over the set of normalised product vectors. Using this norm I introduce an equivalent formulation of the separability problem: $\forall\eta\ \langle\eta|\rho\rangle \leqslant ||\sqrt{\eta}||^2_{S-sup}$. The difficulty here is hidden in the calculation of the $S-sup$ norm. We can get an easy necessary condition of separability if we restrict ourselves to pure η. It leads to a spectral condition of separability - no eigenvalue of a separable state cannot exceed the square of maximal Schmidt coefficient of the related eigenvector. What is interesting, for states diagonal in Bell basis it is also a sufficient condition. Similar restrictions concern sums of eigenvalues of a separable state - then for the right-hand side one has to take the square of the maximal Schmidt coefficient of the subspace spanned by the related eigenvectors. I have shown, that such a criterion can already detect a PPT entanglement.

In the work [17] the authors derive sufficient conditions for eigenvalues of k-Schmidt witness not using the properties of related eigenvectors (like for example their Schmidt coefficients). In the paper [W14] we show, that so defined witnesses cannot detect the PPT entanglement.

The geometry of the set of states of qubit in the language of low-dimensional sections

In the paper [18] the authors considered shapes of sections of the set of qutrit states with surfaces spanned by pairs of Gell-Manm matrices. In response, in the paper [W15] we have shown that the set of all possible sections is easy to parametrise. The shape of a generic section is a third-degree curve on a surface and in some special cases its fragments are curves of the second and first degree. We have shown, where these special cases lie in the set of all sections (the finite set of sections from [18] is contained in this subset). In this subset we have fully characterised all shapes of intersections. The picture showing where these special cases lie in the set of all sections hit the cover of Journal of Physics A.

The dynamics of open quantum systems with a memory kernel

In the open system dynamics, the master equation is generalised introducing time-dependent Lindbladians. Any more general dynamics is called (imprecisely) a non-markovian dynamics. In the paper [W16] we have considered the dynamics of an open system with the reduced dynamics given by a convolution equation with the memory kernel

$$\dot{\rho}(t) = \int_0^t K(\tau)\rho(t-\tau)d\tau.$$

One gets such an equation from the Nakajima-Zwanzig equation (the semi-group part in the equation can be hidden in the atomic part of K). The fundamental question is, which kernels lead to a proper dynamics, i.e. preserving the positivity of the density operator. In the paper we restrict ourselves to commutative kernels for which $[K(t_1), K(t_2)] = 0$ - in any channel there is an independent dynamics. Despite such a simplifying assumption, it is still a hard question. In the paper we give a sufficient condition giving rise to a method of construction of a wide class of such kernels. We derive it doing the Laplace transform of the equation and checking when the transform of ρ will be a *complete monotone* function. Generalising the classical definition of markovianity to the quantum case (a direct generalisation is not possible because there is no conditional probability in non-commutative probability theories) one introduces the definitions of divisibility of the dynamics. For a family of propagators $L_{s,t}$ the dynamics is CP-divisible if for any t the propagator $L_{0,t}$ (which is always completely positive) for any intermediate time s can be represented as the product of two completely positive propagators $L_{0,s}L_{s,t}$. Similarly the dynamics is P-divisible if there always exist a decomposition, but the piecewise propagators suffice to be only positive. We show, that in our construction it is possible to get a CP-divisible dynamic, P-divisible or without divisibility. The last possibility means the existence of flowback of information from the environment to the system. In such a case the environment works as a auxiliary register of the system.

7. Ongoing projects and future plans

Nowadays I work on the dynamics of open quantum systems given by convolution equations and examine the classes of rational CM functions, which allow us to formulate sufficient conditions on the memory kernel, leading to a proper dynamics of the system. The next topic is to go out in this analysis from the class of memory kernels which commute in different moments of time to the most general, fully quantum class.

The next topic I would like to go back concerns the problems of quantum thermodynamics which I worked on in Stockholm with I. Bengtsson and G. Lindblad. The quantum analogon of a heat machine is an open quantum system coupled to (at least) two non-interacting reservoirs, periodically controlled. Such systems are now experimentally accessible what allows to verify the theoretical results and gives insight how the thermodynamics laws work in the limit of small systems.

8. **Leadership and participation in research projects**

   - Co-researcher in grant 3004/B/H03/2007/33 *Quantum entanglement: analysis and classification*
   - Co-researcher in task POKL.04.01.01-00-081/10 *Enhancing didactic potential of UMK in Toruń in mathematical-scientific areas*
   - Co-researcher in grant DEC-2011/03/B/ST2/00136 *Quantum correlations: analysis, detection, dynamics*

9. **Oral communications and participation in conferences**

   Invited lectures

   - Symmetry and structural properties of condensed matter, Myczkowce 2007.IX.5-12
     title: *Spectral properties of entanglement witnesses and separable states*
   - Symmetry and structural properties of condensed matter, Myczkowce 2009.IX.2-9
     title: *Optimality of entanglement witnesses*
   - Geometry of Quantum Entanglement, Centre International de Rencontres Mathématiques (CIRM), Marseille 2012.I.9-13
     title *Exposed positive maps and entanglement witnesses - a sufficient condition*

   Other oral communications

   - Symposium of LFPPI Net, Łódź 13th - 15th april of 2007
     author of the talk *Spectral properties of entanglement witnesses*
   - Symposium of Mathematical Physics, Toruń 11th - 12th june of 2007
     author of the talk *Spectral properties of entanglement witnesses*
   - IV Łódz Symposium of the Network of Laboratories of Physical Foundations of Information Processing, Łódź, 4 april 2008
     author of the talk *Spectral properties of separable states*

- Symposium of the Network of Laboratories of Physical Foundations of Information Processing, Sopot 23-25 april 2009
  author of the talk *Optimality of entanglement witnesses*
- Symposium on Mathematical Physics, Toruń 5-6 june 2009
  author of the talk *Optimality of entanglement witnesses*
- 43 Symposium on Mathematical Physics, Toruń, June 20-22, 2011
  author of the talk *Optimal entanglement witnesses without the property of Hilbert space spanning*

Participation in conferences

- Conference „Quantum entanglement and symetries", Łódź, 29th - 30th november of 2003
- Symposium „Cryptography and Quantum Information", Wrocław/Karpacz, 12th - 17th january of 2004
- Symposium on Mathematical Physics, Toruń 9th - 12th june of 2004
- Symposium „Information and Quantum Mechanics", Wrocław 4th march of 2005
- Symposium on Mathematical Physics, Toruń 17th - 18th june of 2005
- Symposium „Quantum Information & Enginering", Wrocław, 27th january of 2006
- Symposium on Mathematical Physics, Toruń 4th -7th june of 2006
- Symposium of LFPPI Net, Łódź 13th - 15th april of 2007
- Symposium of Mathematical Physics, Toruń 11th - 12th june of 2007
- XXVI Workshop on Geometrical Methods in Physics, Białowieża 29th june – 8th july of 2007
- 10th Workshop: Non-commutative harmonic analysis with applications to probability, Będlewo 6th -12th august of 2007
- Symmetry and structural properties of condensed matter, Myczkowce 5th -12th september of 2007
- Symposium on Mathematical Physics, Toruń 25-28 june 2008
- Foundations of Probability and Physics-5, Växjö, august 24-27 2008
- Italian Quantum Information Science Conference, Camerino 24-29 october 2008
- Symposium of the Network of Laboratories of Physical Foundations of Information Processing, Sopot 23-25 april 2009

- Symposium on Mathematical Physics, Toruń 5-6 june 2009
- Symmetry and structural properties of condensed matter, Myczkowce, 2-9 september 2009
- International Conference on Quantum Information Processing and Communication, Rome, 21-25 september, 2009
- 42 Symposium on Mathematical Physics, Toruń, June 19-22, 2010
- 43 Symposium on Mathematical Physics, Toruń, June 20-22, 2011
- Geometry of Quantum Entanglement, Centre International de Rencontres Mathématiques (CIRM), Marseille, France, January 9-13 2012
- Quantum Theory: Reconsideration of Foundations - 6, Växjö, 11-14 june 2012
- 44 Symposium on Mathematical Physics, Toruń, June 20-24, 2012
- KCIK Symposium, Sopot, May 23-25 2013
- 45 Symposium on Mathematical Physics, Toruń, June 1-2, 2013
- QIP 2014, Barcelona, February 3-7 2014
- The V Jubilee KCIK Symposium, Sopot, May 22-24 2014
- 46 Symposium on Mathematical Physics, Toruń, June 15-17, 2014
- 51 Winter School of Theoretical Physics, Ladek, February 9-14, 2015

10. **Domestic and international cooperation**

- Cooperation with M.Lewenstein i R.Augusiak (ICFO, Barcelona)
- Cooperation with I.Bengtsson i G. Lindblad (Stockholms Universitet, KTH)
- Cooperation with M.Mozrzymas (Wrocław university)
- Cooperation with A.Rutkowski (The National Centre of Quantum Informatics, Sopot)
- Cooperation with S.Maniscalco (Turku university)
- Cooperation with the group of M.Ohya (Tokyo University of Science)

11. **Bibliometrical data**
Number of published articles: 15 + 1 preprints
Google Scholar:

Citations	188	176 (since 2010)
h-index	8	8 (since 2010)
i10-index	8	8 (since 2010)

Web of Science

Sum of the Times Cited	105
Sum of Times Cited without self-citations	82
h-index	6

Bibliography

[1] R. F. Werner *Quantum states with Einstein-Podolsky-Rosen correlations admitting a hidden-variable model* Phys. Rev. A **40**, 4277 (1989)

[2] L. Gurvits *Classical deterministic complexity of edmonds problem and quantum entanglement* Proc. of the 35th ACM symp. on Theory of comp., 10-19, (2003)

[3] S. Gharibian *Strong NP-Hardness of the Quantum Separability Problem* Quantum Information and Computation, Vol. 10, No. 3&4, pp. 343-360 (2010)

[4] B. M. Terhal *Bell inequalities and separability criterion* Phys. Lett. A 271, 319 (2000)

[5] A. Peres *Separability criterion for density matrices* Phys. Rev. Lett. 77(8), 1413 (1996)

[6] K. Chen, L.A. Wu *A matrix realignment method for recognizing entanglement*, Quantum Inf. Comput. 3 193 (2003)

[7] A. C. Doherty, P. A. Parrilo, F. M. Spedalieri *Distinguishing separable and entangled states* Phys. Rev. Lett. 88, 187904 (2002).

[8] A. C. Doherty, P. A. Parrilo, F. M. Spedalieri *A complete family of separability criteria* Phys. Rev. A 69, 022308 (2004)

[9] M. Lewenstein, B. Kraus, J. I. Cirac, P. Horodecki *Optimization of entanglement witnesses* Phys. Rev. A 62, 052310 (2000)

[10] K.-Ch. Ha, S.-H. Kye *Optimality for indecomposable entanglement witnesses* Phys. Rev. A 86, 034301 (2012)

[11] S. Straszewicz *Über exponierte Punkte abgeschlossener Punktemengen* Fund. Math. 24, 139-143 (1935)

[12] M. Marciniak *Rank properties of exposed positive maps* Linear and Multilinear Algebra, 61, Number 7, 970-975 (2013)

[13] P.Horodecki, A.Ekert *Method for Direct Detection of Quantum Entanglement* PRL **89**, 127902 (2002) arXiv:quant-ph/0111064

[14] J.K. Korbicz, M.L. Almeida, J. Bae, M. Lewenstein, A. Acin, Phys. Rev. A **78**, 062105 (2008).

[15] K.-Ch. Ha, S.-H. Kye *The structural physical approximations and optimal entanglement witnesses* J. Math. Phys. 53, 102204 (2012)

[16] A. Sanpera, D. Bruss, M. Lewenstein *Schmidt number witnesses and bound entanglement* Phys. Rev. A 63 050301 (2001) arXiv:quant-ph/0009109

[17] D. Chruściński, A Kossakowski *Spectral conditions for positive maps* Comm. Math. Phys. 290, 1051 (2009)

[18] S. K. Goyal, B. N. Simon, R. Singh, S. Simon *Geometry of the generalized Bloch sphere for qutrit* arXiv:1111.4427